PERGAMON

International Journal of Multiphase Flow 25 (1999) 559–574

International Journal of Multiphase Flow

The sedimentation velocity of dilute suspensions of nearly monosized spheres

R. Di Felice

Istituto di Ingegneria Chimica e di Processo "G.B. Bonino", Universita' degli Studi di Genova, via Opera Pia, 15, 16145, Genova, Italy

Received 1 May 1997; received in revised form 5 December 1998

Abstract

The sedimentation velocity of dilute suspensions of spheres, with particle volume concentration up to 5%, has been experimentally measured for a variety of solid–liquid systems covering a large range of Reynolds numbers (from 0.01 to 1000). For the systems experimentally investigated, it was found that the settling velocity was a linear function of the particle volume concentration, the dependency being less marked as the Reynolds number increased. A comparison between dilute and concentrated systems behaviour suggested a possible improvement of the Richardson–Zaki equation.

Keywords: Hindered settling velocity; Multiparticle dilute suspension; Viscous flow regime; Inertial flow regime

1. Introduction

Measurements of the sedimentation velocity of solid particles is of both academic interest and practical relevance. Researchers are interested in the settling velocity as a means of confronting theoretical predictions for the fluid dynamic description of two-phase systems with experimental observations, or in order to estimate solid–fluid interaction forces in the fluid dynamic regimes for which direct theoretical approaches are unavailable. On the other hand, settling velocity is the key parameter for industrial units in which particles and fluid need to be separated, such as in the clarification of waste water or in the mining industry processes.

The behaviour of a single solid sphere placed in a Newtonian fluid is well known: it will fall, after a short acceleration time, at its terminal settling velocity, u_t; for a large expanse of fluid, the terminal velocity is easily calculated from the solid and fluid physical characteristics

0301-9322/99/$ - see front matter
PII: S0301-9322(98)00084-6

through the following equation:

$$u_t = \left[\frac{4d(\rho_p - \rho)g}{3C_D\rho} \right]^{0.5} \tag{1}$$

where C_D is the drag coefficient, for which empirical expressions of varying degrees of complexity and accuracy are available for all flow regimes. A reasonable compromise is the expression attributed to Dallavalle:

$$C_D = \left[0.63 + \frac{4.8}{Re_t^{0.5}} \right]^2 \tag{2}$$

For a particle in a suspension of solids the settling velocity will be smaller than the terminal velocity and will depend on the particle volume concentration, C. The upwards flux of fluid, compensating for the downwards flux of the solids and of the fluid dragged down by it, is the main factor giving rise to the reduction of the settling velocity for multiparticle suspensions. The actual dependency of the settling velocity on the particle volume concentration can be estimated theoretically only for dilute suspensions in the viscous flow regime, where the integration of the Navier–Stokes equations of motion is possible, although, as will be made clearer later, conflicting different dependencies on the particle volume concentration have been proposed. On the other hand, when inertial effects are no longer negligible (the intermediate and inertial flow regimes), the hindering effect of the particle volume concentration on the settling velocity can only be determined experimentally—although, surprisingly, no data appear to have been published in the open literature for dilute suspensions.

The main aim of this paper is to report experimental data on the dependency of the settling velocity on particle concentration in dilute suspensions (we will consider "dilute" to refer to particle volume concentrations up to 5%).

2. Previous relevant works

2.1. Low Reynolds number regime

We will consider first the case of sedimenting dilute suspensions in the viscous flow regime. The reason for doing so is twofold: the availability of theoretical predictions, obtained in a exact way by integrating the Stokes equations of motions; and an abundance of experimental evidence.

As discussed by Davis and Acrivos (1985), the theoretical efforts can be divided in two groups. In the first one the hindering effect of the surrounding particles is found to be proportional to the particle volume concentration to the first power: Batchelor (1972) studied the settling velocity of randomly dispersed suspensions of spheres and found

$$u = u_t(1 - nC) \tag{3}$$

where n was 6.55 for perfectly monosized suspensions and, for the more realistic case of a suspension made up from particles of the same density and slightly dispersed size, n was a function of the Peclet number (this dimensionless parameter quantifies the magnitude of the motion due to gravity relative to the motion due to Brownian diffusion, that is to say that for system with high Peclet number gravity motion is the important one). For negligible interparticle forces, Batchelor and Wen (1982) suggested for n a value close to 5.5 for suspensions with very large Pe number and a value close to 6.5 for suspensions with very small Pe number.

The second group of theoretical approaches predicts the hindered settling velocity to be a function of the particle volume concentration to the power 1/3. All the cell models and the particle fixed spatial arrangement models fall in this category; we consider here, for comparison purpose, only the relationship obtained by Happel and Epstein (1954) for dilute suspensions:

$$u = \frac{u_t}{1 + 15C^{1/3}} \tag{4}$$

Other workers have reported similar expressions, the only difference being the numerical constant multiplying the variable C in Eq. (4) which, depending on the assumptions made, varies in the range 1–2.1 (Barnea and Mizrahi, 1973).

Settling velocities (made dimensionless by dividing them by u_t) as predicted by Eqs. (3) and (4) are plotted in Fig. 1 as functions of particle volume concentration. It would seem that, given the marked diversity of the two curves, a basically simple experiment would be able to distinguish between the two theoretical results, i.e. to tell us if the hindering effect in practical cases is a function of C to the power 1 or a function of C to the power 1/3. Unfortunately, this is not the case: Fig. 2 depicts selected experimental values of hindered settling velocity in which the data split into two quite separate groups. While some experiments suggest that Eq. (3) is correct (Cheng and Schachman, 1955; Ham and Homsy, 1988; Davis and Birdsell, 1988; Al-

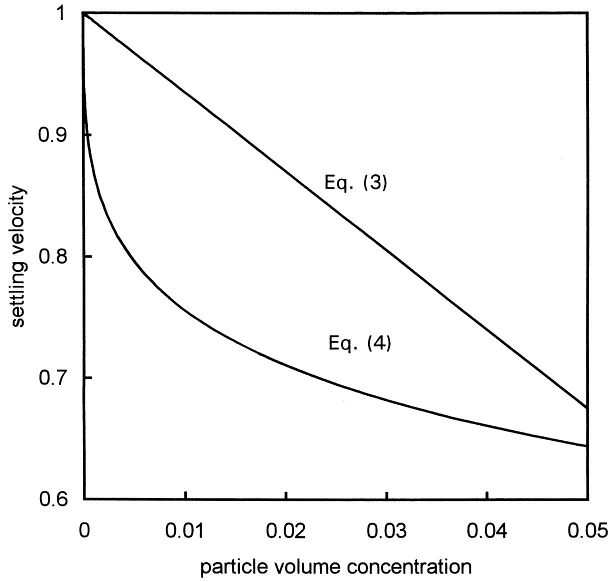

Fig. 1. Dimensionless settling velocity at low Reynolds numbers as predicted by Eqs. (3) and (4).

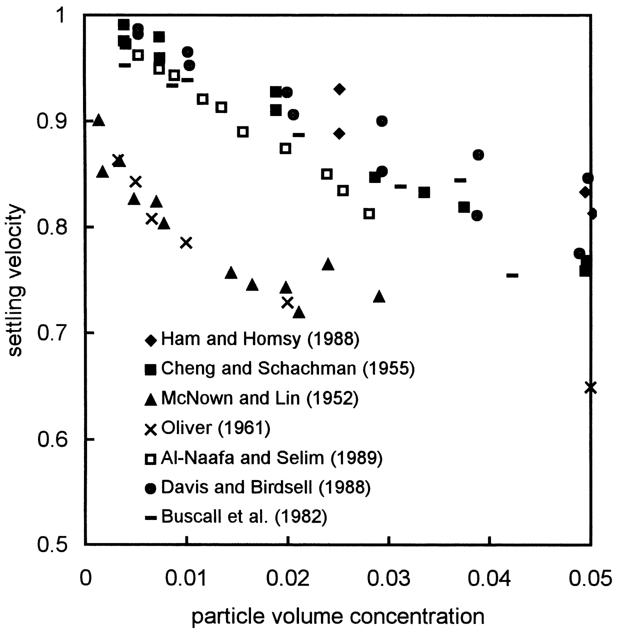

Fig. 2. Experimentally measured settling velocity at low Reynolds numbers.

Naafa and Selim, 1992; Buscall et al., 1982), others support Eq. (4) (McNown and Lin, 1952; Oliver, 1961).

2.2. High Reynolds number regime

The lack of exact theoretical treatments outside the viscous flow regime leaves room for other approaches, with varying degrees of empiricism. An example for the inertial flow regime has been given recently by Di Felice et al. (1995), who made use of the work of Newton (1687) on the effect of the wall on the drag force on an isolated particle to derive an expression for the sedimentation velocity of multiparticle suspensions at high Reynolds number (including this time the effect of buoyancy, as demonstrated in Appendix A):

$$u = u_t\left(\varepsilon^{2.5}\left(\frac{1+\varepsilon}{2}\right)^{0.5}\right) \approx u_t\varepsilon^{2.7} \tag{5}$$

For the dilute condition of the present investigation, Eq. (5) is equivalent to:

$$u = u_t(1 - 2.7C) \tag{6}$$

To our knowledge, no experimental data is available on the sedimentation velocity of dilute suspensions at high Reynolds numbers.

2.3. Intermediate Reynolds number regime

As in the previous case, predictive correlations can only by of empirical or semi-empirical nature. The Richardson and Zaki (1954) relationship, derived originally in an empirical manner from experimental data on sedimentation and fluidization at high particle volume concentrations (generally greater than 10%), is easily the most popular:

$$u = u_t(1 - C)^n \tag{7}$$

and it could be used for dilute suspensions too; in this case Eq. (7) reduces to Eq. (3). The value of the parameter n is not constant any more but a function of flow regime and has been given by Rowe (1987) in a compact form

$$\frac{4.7 - n}{n - 2.35} = 0.175 Re_t^{0.75} \tag{8}$$

Again, no specific data on the sedimentation velocity of dilute suspensions in this specific flow regime appear to have been published in the open literature.

2.4. The effect of the column size

The settling velocity depends also on the size of the container where the sedimentation is

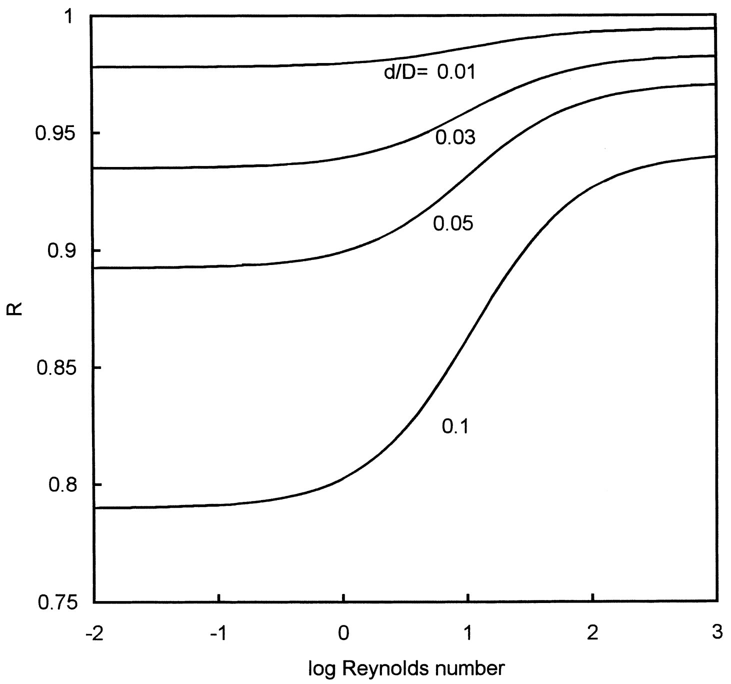

Fig. 3. Bounded to unbounded terminal settling velocity ratio function of the Reynolds number as calculated from Eqs. (9) and (10).

taking place, and more precisely for circular container on the ratio between particle and container diameter, d/D: the smaller the container cross section area compared to the particle, the more important the effect is.

For a single particle the retarding effect of the wall is well known and documented: recently Di Felice (1996) proposed a relationship applicable at any flow regime. If R is the ratio between bounded and unbounded settling velocity then

$$R = \left[\frac{1 - d/D}{1 - 0.33d/D}\right]^{a} \tag{9}$$

with α given by

$$\frac{3.3 - a}{a - 0.85} = 0.1Re_{\mathrm{t}} \tag{10}$$

Fig. 3 depicts calculated R for various values of the ratio d/D. The reduction in settling velocity is appreciable, i.e. experimentally distinguishable, if the ratio d/D is greater than 0.05.

For multiparticle systems the effect of the container wall is not quantitatively known. However, intuition suggests that container cross section area is less and less important as the particle volume concentration increases. This has been experimentally demonstrated for the settling velocity of spheres in viscous flow regime (Di Felice and Parodi, 1996). It was shown there that for particle concentration greater than 0.1, settling velocities were not affected even for value of d/D over 0.2.

3. Experimental

In this work batch sedimentation particle velocity has been investigated experimentally for dilute suspensions of nearly monosized spheres in Newtonian fluids for a wide range of Reynolds numbers: we studied systems at Re_{t} up to 1000 for particle volume concentrations varying from 0 to about 5%.

Davis and Birdsell (1988) have discussed the practical difficulties arising when settling velocities of dilute suspensions are to be measured. Typical problems concern the diffuse suspension–clear fluid interface at low solid concentrations that is due to particle diffusion and to particle size spread and, in making the data dimensionless, uncertainty of up to 10% in the experimental or theoretical determination of the single particle terminal settling velocity. Obviously the use of spheres of well-defined diameter is an effective means of reducing uncertainty in settling velocity determinations; these spheres are to be found in practice either with very small diameter (of the order of 1 μm or less) or rather large (1)mm or more). In contrast, the most easily obtainable material, commonly used in fluidization and sedimentation laboratory experiments, has a medium size in the range of 0.1–1 mm in diameter, and a significant size spread.

In the work reported here we used large, nearly monosized, particles which offered two specific advantages: the unambiguous experimental determination of the terminal settling velocity, and the possibility of following by naked eye the sedimentation of a single particle.

Three types of solids were used: acetate balls of 3 and 4.9 mm in diameter, with a density of 1280 kg/m^3; and *gunballs* 5.9 mm in diameter with a density of 1050 kg/m^3. For all the three solid types, the maximum deviation of each particle diameter from the average value was less than 0.05 mm. The fluids were water solutions of glycerol, with glycerol content varying from 0 to 88% by weight, so that fluid density ranged from 1000 to 1230 kg/m^3 and fluid viscosity ranged from 0.001 to 0.174 kg/m/s. The fluid characteristics are at 20°C, the temperature at which all the measurements were made. In addition, a single run was carried out with a completely different system: lead shot (1.7 mm in size and 11300 kg/m^3 in density) in silicon oil (density of 955 kg/m^3 and viscosity of 1.3 kg/m/s): this was done to test whether the particle density to fluid density ratio would show any effect. In the event, the results for this latter system was in perfect line with all the others and therefore are shown grouped with the other data.

The experiments were carried out using two different methods. The first one was used for slowly settling systems and a procedure similar to that described at length by Di Felice and Parodi (1996) was adopted: a cylindrical column with a diameter of 107 mm and 500 mm tall was filled with known amount of fluid and solid, which were then thoroughly mixed by rotating the tube axially and laterally. The settling velocity of the falling solid–clear fluid interface was then measured with a stopwatch over a distance of 50 or 100 mm. For systems possessing high settling velocities this procedure was not feasible. For these cases, the starting homogeneous particle–fluid suspension was obtained by circulating fluid and particles in a circulating liquid fluidized bed basically identical to the one described in a previous paper (Gibilaro et al., 1988). The test section had an internal diameter of 140 mm and was 1800 mm tall. It generally took no more than one minute of vigorous suspension circulation to achieve a satisfactory homogeneity: at that point the pump was switched off, the butterfly valve in the loop closed, and the settling velocity measured over a 600 mm length distance. In order to maintain the temperature in the range 20 ± 0.1°C the first, smaller column was immersed in a thermostatic bath; this method was not applicable for the second larger rig. In this case the temperature of the fluid was continuously monitored: as soon as an increase above the upper threshold was observed, some of the fluid was withdrawn and replaced with fresh, cooler, liquid sending the temperature down towards the lower threshold.

Table 1 summarizes the physical characteristics of the systems used. From the table it will be seen that the ratio d/D varied up to a value of 0.0458. This makes the retarding effect of the container wall, both for the single and multiparticle suspensions, negligible for the present investigation and therefore it has been ignored in the subsequent analysis of the data.

Davis and Hassen (1988) have pointed out that the interface of a settling suspension undergoes two contrasting phenomena: a self-sharpening effect and a spreading due to particle diffusion. In the present investigation the solid–fluid interface always looked quite sharp, and any spreading, if present, could not be detected with the simple device (i.e. the naked eye) utilized.

Unhindered settling velocities were measured experimentally by dropping a single particle at the axis of the same tube in which the sedimentation experiments were carried out.

Table 1
System physical characteristics employed in the present investigation

Solid		Fluid		Column	Terminal settling velocity		
Density (kg/m^3)	Diameter (mm)	Density (kg/m^3)	Viscosity (kg/m/s)	Diameter (mm)	Calculated Eqs (1) and (2) (mm/s)	Calculated Eqs. (9) and (10) (mm/s)	Experimental (mm/s)
1280	4.9	1230	0.174	107	3.6	3.3	3.7
1280	4.9	1203	0.049	107	15.6	14.2	15.2
1280	4.9	1181	0.023	107	31.8	29.6	29.9
1280	4.9	1154	0.010	140	57.2	55.3	52.3
1280	4.9	1134	0.007	140	72.5	70.7	67.7
1280	4.9	1115	0.005	140	87.5	85.3	90.4
1280	4.9	1073	0.003	140	120.1	117.4	135.3
1280	4.9	1000	0.001	140	169.0	165.1	189.5
1280	3.0	1203	0.049	107	6.7	6.3	6.6
1280	3.0	1181	0.023	107	15.4	14.6	14.4
1050	5.9	1000	0.001	140	72.0	70.5	62.0
11300	1.7	955	1.300	107	12.6	12.2	12.5

4. Results and discussion

A typical example of experimental result is shown in Fig. 4. The symbols represent the average of all the measurements for one system condition, generally about 20–30 experimental measurements and never less than 15. The measured unhindered terminal settling velocities are

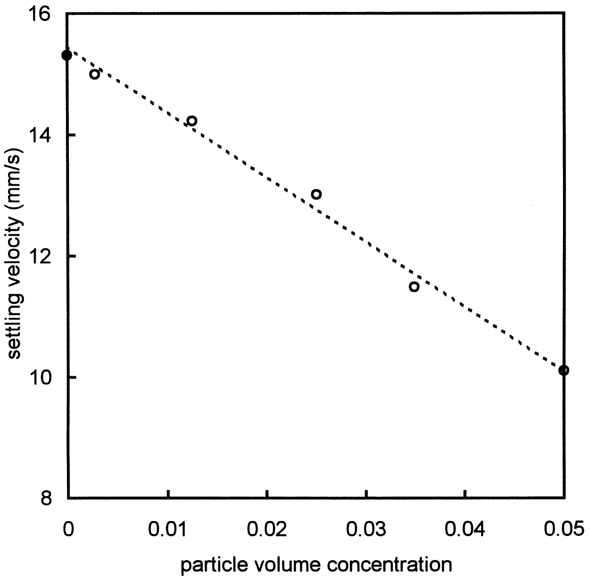

Fig. 4. Measured settling velocities function of particle volume concentration. The case reported here refers to 4.9 mm acetate balls in a 78% glycerol solution.

compared in Table 1 with those calculated by means of Eqs. (1), (2), (9) and (10). The difference between the two set of data underlines the uncertainty in the estimation of u_t from the physical characteristics of the systems. Fig. 4, and indeed all the other systems investigated in the present work, indicates quite unequivocally that the influence of the particle concentration on the settling velocity is well described by a simple linear relationship. Consequently, all the data were fitted by a relation of the type

$$u = a + bC \tag{11}$$

(the single particle results were not included in the fitting exercise).

Fig. 5 compares the values of a from the minimization routine with the measured u_t; the two data sets are practically coincident; this implies that the linear extrapolation of the suspension settling velocities at particle concentration equal to zero is indeed represented by the experimental single particle terminal settling velocity. Therefore Eq. (3), and consequently Eq. (7), satisfactorily represent the present experimental investigation. Fig. 6 depicts the n values obtained from the minimizing routine ($n = -b/a$), which are well summarized by

$$\frac{6.5 - n}{n - 3} = 0.1 Re_t^{0.74} \tag{12}$$

4.1. Low Reynolds number regime

For small values of the particle terminal Reynolds number, $Re_t < 1$, the values of the coefficient n from the experiments are practically constant, within normal fluctuations, at about

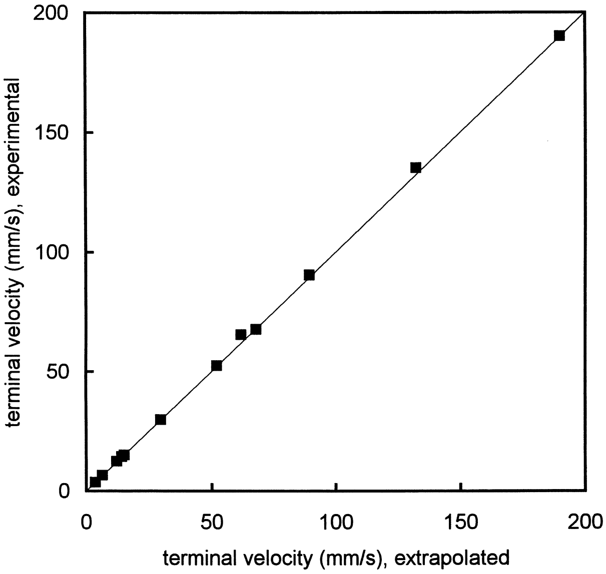

Fig. 5. Comparison of extrapolated settling velocity to particle concentration equal to zero against measured single particle settling velocity.

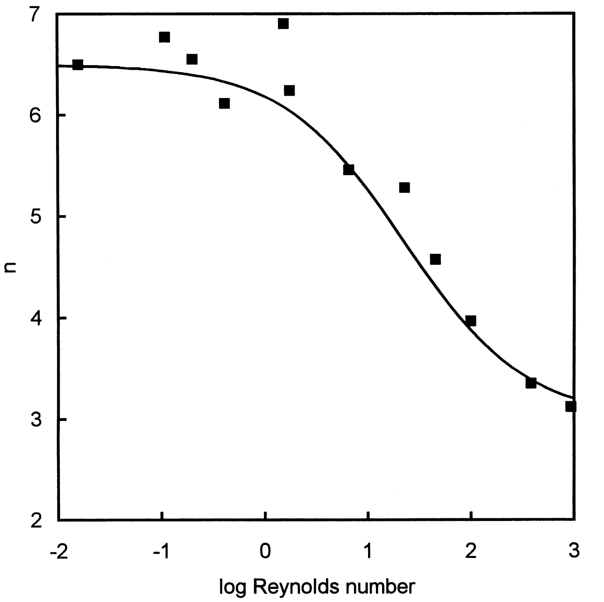

Fig. 6. Experimental values of n as a function of Reynolds number (points) and fitting curve (continuous line).

6.5. This value is somewhat higher than the theoretical prediction of 5.5 (Batchelor and Wen, 1982) and broadly in agreement with reported experimental findings: n ranging from 5.1 (Cheng and Schachman, 1955) to 7.5 (Al-Naafa and Selim, 1992). It is not certain, however, if a comparison with the present work can be made straight away as the solids utilized in this work are very large and so is the Peclet number, whereas the mentioned works concern the settling of very small, sub-micron, particles with small Peclet number.

Buyevich (1995) has shown that when interface settling velocities are measured as in the present work, the coefficient n should be higher than that obtained when actual particle settling velocities are measured, due to particle velocity fluctuations: he suggested a value as high as 8.3 and this could explain the deviation from the 5.5 derived from Batchelor's work.

Rather than the actual value of n the most relevant finding of this work is the linear relationship linking settling velocity and particle volume concentration, in line with Batchelor's theoretical approach. The analysis of previous works was not clear in this respect, as already illustrated in Fig. 2. Why there is such a discrepancy in reported data is an open question. It is worth noting that the data in Fig. 2 supporting Eq. (4) were obtained with medium particles (diameter in the range 0.1–1 mm). Davis and Acrivos (1985) speculated that different particle size could be related to different suspension microstructure, and consequently produce different settling velocity behaviour, but no evidence has been found yet to support this argument.

The data in Fig. 2 could be actually drawn together by assuming that some error has been made in the determination of the terminal settling velocity. Arbitrary correction of u_t would result in a translation of the data along the y-axis and it is easily seen that such movement could make all the points to fall on roughly the same line. This artifice, however, would not really help in choosing between Eqs. (3) and (4), but would serve to emphasize the importance of the correct determination of the unhindered settling velocity. In the studies published so far,

u_t has either been calculated through Eq. (1), with all the problems deriving from the exact knowledge of the system physical parameters, or, even worse, has been obtained by extrapolating suspension settling velocities, neglecting the fact that a correct extrapolation requires a priori knowledge of the characteristic settling law. In the present study the use of very large solid particles has enabled us to overcome this problem.

4.2. High Reynolds number regime

For the system possessing the highest Reynolds number ($Re_t = 980$) the experimental value of n found was 3.1, which compare quite satisfactorily with the value of 2.7 obtained from the simplified theory presented here for the fully inertial flow regime.

The difference in behaviour between suspensions in viscous and inertial flow regime can be explained qualitatively if we consider the main factor retarding the settling of the particles, i.e. the counterflow of fluid needed to balance the downwards flux of the solid particles and of the fluid dragged down by them. If we assume that a particle is settling with f time, its volume of fluid attached, and if we assume the slip velocity between particle and fluid to be constant at u_t, then a simple material balance leads to:

$$u = u_t[1 - C(1 + f)] \tag{13}$$

Batchelor (1972) estimated f to be equal to 4.5 for a suspension in viscous flow; at the other extremity of flow regime Kowe et al. (1988) suggested a value of 0.5 for f. The corresponding

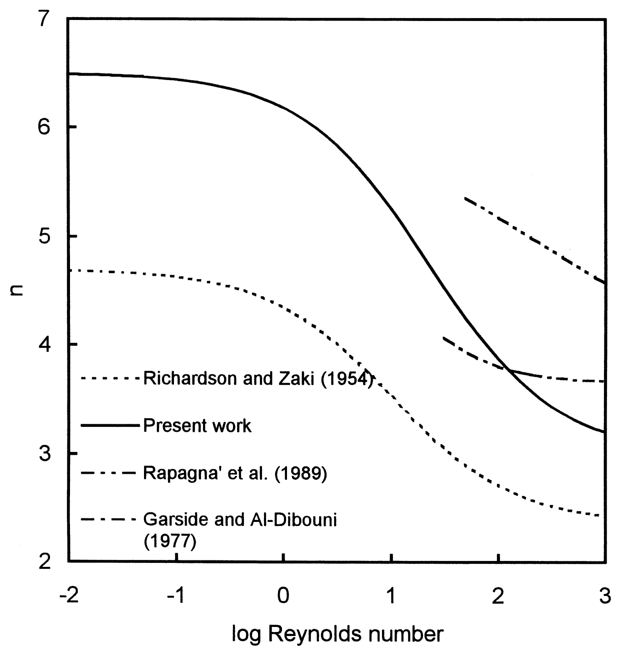

Fig. 7. The parameter n as a function of the system Reynolds number; findings of the present work compared with published correlations.

estimates for n would be then 5.5 and 1.5, respectively, indicating that, taking into account even only this single contribution, the observed dependency on the Reynolds number can be justified.

4.3. Intermediate Reynolds number regime

Outside the viscous flow regime, the coefficient n decreases as Reynolds number increases. There is no corresponding work with which to compare the present results; the nearest case we can use involves data for fluidization at high voidages. Typical examples have been reported by Garside and Al-Dibouni (1977) and by Rapagna' et al. (1989). Fig. 7 compares the results with that mentioned before and, although there is no perfect numerical agreement, the trend is well respected.

Fig. 7 also compares the present result with the well known work of Richardson and Zaki (1954) which regarded, it should be stressed again, *concentrated* sedimenting and fluidizing systems. Examination of Fig. 7 shows the two experimental findings to follow practically the same law as the Reynolds number changes, the only difference being that the values of n for dilute systems are some 1.5 times the values of n for concentrated systems. If the coefficient n is effectively higher under dilute conditions it would mean that the Richardson and Zaki equation for concentrated systems would be better written:

$$u = ku_t(1 - C)^n \tag{14}$$

with k smaller than 1. The corresponding voidage–velocity relationship covering the whole range of suspension voidages is schematically depicted in Fig. 8. There is little doubt that Fig. 8 is a good representation of fluidized bed expansion characteristics for large, low density,

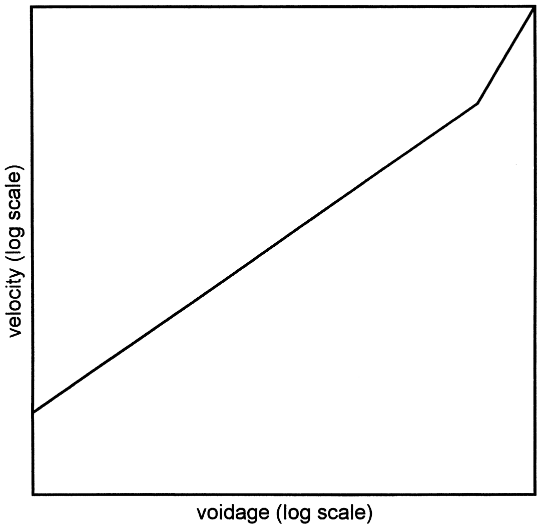

Fig. 8. Schematic velocity–voidage relationship predicted from the present work on dilute systems and published work on concentrated systems.

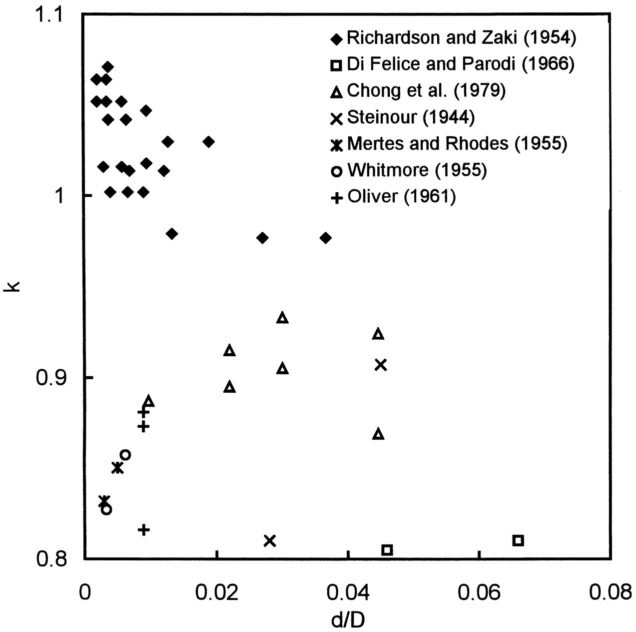

Fig. 9. Experimental values of the parameter k from data on concentrated systems as a function of the particle to column diameter ratio (Richardson and Zaki, 1954; Di Felice and Parodi, 1966; Chong et al., 1979; Steinour, 1944; Mertes and Rhodes, 1955; Whitmore, 1955; Oliver, 1961).

solids (such as plastic or soda glass) at Reynolds number greater that 100, as the work of Riba and Couderc (1977) and Rapagna' et al. (1989) have clearly shown, and whether this is also true for any solid–fluid system is yet to be resolved. This would involve investigating systems with a higher solid to fluid density ratio than the one employed here. Fig. 9 presents, nonetheless, a somewhat interesting picture. There experimental values of k are reported for concentrated sedimenting suspensions of medium and large solids (k has been reported as a function of the particle to wall diameter ratio to show that this parameter has little or no effect on concentrated suspension behaviour, Di Felice and Parodi, 1996). It is rather surprising that Richardson and Zaki (1954) were the only ones to report values of k close to or greater than 1 (it must be said that Richardson and Zaki compared calculated and extrapolated terminal settling velocity using logarithmic values: in this case the two sets of data were practically coincident). All the other works reported values of k well below 1, in the region of 0.8–0.9, in agreement with the findings of the present investigation.

5. Conclusions

The present experimental investigation has verified that the settling velocity of dilute suspensions of nearly monosized spheres depends linearly on the particle volume concentration. This is in agreement with Batchelor's theoretical predictions for the viscous flow regime and with a simplified semi-theoretical approach presented in a previous paper by Di Felice et al. (1995) for the inertial flow regime.

The present results also suggest that the extrapolation of the Richardson and Zaki equation to voidage unity should fall short of the single particle terminal velocity. As a consequence, the equation for the fluidization and sedimentation velocity of concentrated suspension should take this into account.

Acknowledgements

The financial support of the University of Genova is gratefully acknowledged.

Appendix A

For a given fluid–multiparticle system the drag force acting on a particle can be written as

$$F_D \propto f(\varepsilon) C_D u^2 \tag{A1}$$

where $f(\varepsilon)$ is the voidage function, defined elsewhere (Di Felice, 1994). Let us assume that the drag force is known and that we want to obtain the sedimenting characteristics law. The drag force must balance the effective weight of the particle (weight–buoyancy). As is well known, there are different views on the magnitude of the buoyancy force. If we say that the buoyancy is obtained as the particle is displacing only the fluid then the effective weight is

$$W_C = V_p(\rho_p - \rho)\boldsymbol{g} \tag{A2}$$

On the other hand, if we say that the particle is displacing the suspension

$$W_G = V_p(\rho_p - \rho)\boldsymbol{g}\varepsilon \tag{A3}$$

By considering terminal condition we also have

$$W_C = \frac{W_G}{\varepsilon} = F_{Dt} \propto C_{Dt} u_t^2 \tag{A4}$$

If Eq. (A2) is correct, then equating Eq. (A1) with Eq. (A2) using Eq. (A4):

$$\frac{u}{u_t} = \left(\frac{C_{Dt}}{C_D f(\varepsilon)}\right)^{0.5} \tag{A5}$$

Analogously, if Eq. (A3) is the correct effective weight as the present author believes, then

$$\frac{u}{u_t} = \left(\frac{C_{Dt}\varepsilon}{C_D f(\varepsilon)}\right)^{0.5} \tag{A6}$$

The two relationships, Eqs. (A5) and (A6), differ by a factor which, at low Reynolds number is equal to ε and at high Reynolds number equal to $\varepsilon^{0.5}$. Di Felice et al. (1995) in obtaining their relationship for sedimenting suspension in inertial flow regime,

$$u = u_t\left(\varepsilon^2\left(\frac{1+\varepsilon}{2}\right)^{0.5}\right) \approx u_t\varepsilon^{2.2} \tag{A7}$$

did not consider the effective weight as given by Eq. (A3). Introducing this modification, Eq. (6) rather than Eq. (A7) is obtained.

References

Al-Naafa, M.A., Selim, M.S., 1992. Sedimentation of monodisperse and bidisperse hard-sphere colloidal suspensions. AIChE J. 38, 1618–1630.

Barnea, F., Mizrahi, J., 1973. A generalized approach to fluid dynamics of particulate system. Part I. General correlation for fluidization and sedimentation in solid multiparticle systems. Chem. Engng J. 5, 171–189.

Batchelor, G.K., 1972. Sedimentation in a dilute dispersion of spheres. J. Fluid Mech. 52, 245–268.

Batchelor, G.K., Wen, C-S., 1982. Sedimentation in a dilute polydisperse system of interacting spheres. Part 2. Numerical results. J. Fluid Mech. 124, 495–528.

Buscall, R., Goodwin, J.W., Ottewill, R.H., Tadros, ThF, 1982. The settling of particles through Newtonian and non-Newtonian media. J. Colloid Interface Sci. 85, 78–86.

Buyevich, YuA, 1995, Particle distribution in shear and pressure-driven suspension flow. The 1995 ASME International Mechanical Engineering Congress and Exposition, 12–17 November, San Francisco, CA, pp.107–119.

Cheng, P.Y., Schachman, H.K., 1955. Studies on the validity of the Einstein viscosity law and Stokes' law of sedimentation. J. Polym. Sci. 16, 19–30.

Chong, Y.S., Ratkowsky, D.A., Epstein, N., 1979. Effect of particle shape on hindered settling in creeping flow. Powder Technol. 23, 55–66.

Davis, R.H., Acrivos, A., 1985. Sedimentation of noncolloidal particles at low Reynolds numbers. A. Rev. Fluid Mech. 17, 91–118.

Davis, R.H., Birdsell, K.H., 1988. Hindered settling of semidilute monodisperse and polydisperse suspensions. AIChE J. 34, 123–129.

Davis, R.H., Hassen, M.A., 1988. Spreading of the interface at the top of a slightly polydisperse sedimenting suspension. J. Fluid Mech. 196, 107–134.

Di Felice, R., 1994. The voidage function for fluid–particle interaction systems. Int. J. Multiphase Flow 20, 153–159.

Di Felice, R., 1996. A relationship for the wall effect on the settling velocity of a sphere at any flow regime. Int. J. Multiphase Flow 22, 527–533.

Di Felice, R., Parodi, E., 1996. Wall effects on the sedimentation velocity of suspensions in viscous flow. AIChE J. 42, 927–931.

Di Felice, R., Gibilaro, L.G., Foscolo, P.U., 1995. On the hindered settling velocity of spheres in the inertial flow regime. Chem. Engng Sci. 50, 3005–3006.

Garside, J., Al-Dibouni, M.R., 1977. Velocity–voidage relationships for fluidization and sedimentation in solid–liquid systems. Ind. Engng Chem. Process Des. Dev. 16, 206–214.

Gibilaro, L.G., Di Felice, R., Foscolo, P.U., 1988. A circulating liquid fluidised bed. Chem. Engng Sci. 43, 2901–2903.

Ham, J.M., Homsy, G.M., 1988. Hindered settling and hydrodynamic dispersion in quiescent sedimenting suspensions. Int. J. Multiphase Flow 14, 533–546.

Happel, J., Epstein, N., 1954. Viscous flow in multiparticle systems: cubical assemblage of uniform spheres. Ind. Engng Chem. 46, 1187–1194.

Kowe, R., Hunt, J.C.R., Hunt, A., Couet, B., Bradbury, L.J.S., 1988. The effect of bubbles on the volume fluxes and the pressure gradients in unsteady and non-uniform flow of liquids. Int. J. Multiphase Flow 14, 587–606.

McNown, J.S., Lin, P.N., 1952, Sediment concentration and fall velocity. Proceedings of Second Midwestern Conference on Fluid Mechanics, Ohio State University, pp. 401–411.

Mertes, T.S., Rhodes, H.B., 1955. Liquid–particle behavior (part I). Chem. Engng Prog. 51, 429–432.

Newton, I., 1687. Principia Book II, Prop. XXXIX, Theor. XXXI.

Oliver, D.R., 1961. The sedimentation of suspensions of closely-sized spherical particles. Chem. Engng Sci. 15, 230–242.

Rapagna', S., Di Felice, R., Gibilaro, L.G., Foscolo, P.U., 1989. Steady-state expansion characteristics of monosize spheres fluidised by liquids. Chem. Engng Commun. 79, 131–140.

Riba, J.P., Couderc, J.P., 1977. Expansion de couches fluidisées par des liquides. Can. J. Chem. Engng 55, 118–121.

Richardson, J.F., Zaki, W.N., 1954. Sedimentation and fluidisation. Part I. Trans. Inst. Chem. Engrs 32, 35–53.

Rowe, P.N., 1987. A convenient empirical equation for estimation of the Richardson–Zaki exponent. Chem. Engng Sci. 43, 2795–2796.

Steinour, H.H., 1944. Rate of sedimentation—nonflocculated suspension of uniform spheres. Ind. Engng Chem. 36, 618–624.

Whitmore, R.L., 1955. The sedimentation of suspensions of spheres. Br. J. Appl. Phys. 6, 239–245.